1-14-2014

Issues of Effective Field Theories with Resonances

Jambul Geglia
Ruhr-Universitat Bochum

Physics of Excited Nucleons (NSTAR 2013)
International Journal of Modern Physics: Conference Series
Vol. 26 (2014) 1460094 (7 pages)

DOI: 10.1142/S2010194514600945

ISSUES OF EFFECTIVE FIELD THEORIES WITH RESONANCES*

JAMBUL GEGELIA

Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany
and
Tbilisi State University, 0186 Tbilisi, Georgia
Jambul.Gegelia@tp2.ruhr-uni-bochum.de

GEORGE JAPARIDZE

Clark Atlanta University, Atlanta, Georgia 30314, USA

Published 14 January 2014

We address some issues of renormalization and symmetries of effective field theories with unstable particles - resonances. We also calculate anomalous contributions in the divergence of the singlet axial current in an effective field theory of massive $SU(N)$ Yang-Mills fields interacting with fermions and discuss their possible relevance to the strong CP problem.

Keywords: Renormalization; resonances; anomaly.

PACS numbers: 11.10.Gh, 03.70.+k, 11.30.Er, 11.40.−q

1. Introduction

It is widely accepted that quantum chromodynamics (QCD) is the correct quantum field theory (QFT) of strong interaction. If this is the case then all scattering processes of strongly interacting particles can be described by the S-matrix of QCD. Unstable states, resonances, are manifested as poles in the S-matrix. The low-energy effective field theory (EFT) of strong interaction, chiral perturbation theory, describes the S-matrix of QCD in terms of effective degrees of freedom. Resonances are often represented by corresponding fields in this formalism. Hadronic EFT with resonances is a QFT including unstable particles. Renormalization is a non-trivial issue in chiral EFT. (Extended) on-mass-shell scheme[1–3] proved to be very useful in baryonic sector of chiral EFT. Generalization of the on-mass-shell scheme for QFT with unstable particles is called complex-mass scheme (CMS).[4–6] Below we discuss some non-trivial issues encountered when using CMS.

The modern point of view treats the Standard Model (SM) as an effective field theory (EFT) in the framework of which traditional renormalizability is replaced by the requirement that all divergences are absorbed in the redefinition of an infinite number of parameters of the effective Lagrangian.[7] Effective field theory possesses controllable predictive power for energies lower than some large scale. As a rule the principle of gauge invariance is taken as a starting point in constructing field-theoretical models. Since electromagnetic and gravitational forces are long-range, they have to be described by gauge theories.[7] On the other hand, the strong and weak interactions are short-ranged and therefore, despite enormous success of the SM, it is worthwhile to investigate alternative scenarios with massive vector bosons. In Ref.[8] it has been shown that the most general leading-order consistent EFT Lagrangian of a self-interacting triplet of massive vector bosons is the gauge-invariant Yang-Mills Lagrangian up to globally invariant mass term. Moreover, considering an $SU(3)$ globally invariant EFT of massive vector bosons interacting with fermions and demanding the consistency with constraints and the perturbative renormalizability in the sense of EFT, it is easy to demonstrate that the leading order effective Lagrangian exactly coincides with the $SU(3)$ locally invariant Yang-Mills Lagrangian with additional globally invariant mass term. In this contribution we address the issue of anomalies of the singlet axial current in an EFT of massive $SU(N)$ Yang-Mills fields interacting with fermions. We consider a general case so that our results can be applied to low-energy EFT of strong interactions as well as to an EFT which could serve as a theory of quarks and gluons.

2. The Complex-Mass Scheme

The complex-mass scheme[4–6] is a generalization of the on-mass-shell scheme to unstable particles. In QFT the renormalization can be carried out by splitting bare parameters and fields into renormalized quantities and counter-terms.Similarly to on-mass-shell scheme in CMS renormalized masses are taken equal to poles of dressed propagators. CMS has found many applications in the Standard Model. It has also been applied in hadronic EFT.[9–14]

Perturbative unitarity in CMS has been demonstrated in one-loop order in Ref.[15] Note that one might encounter problems in verifying perturbative unitarity when the complex renormalized couplings are used. Let us demonstrate this on an example of a toy model for the S-matrix

$$S = e^{i\,g}$$

with a real parameter g. Defining $S = 1 + i\,T$, the condition $S^\dagger S = 1$ reduces to

$$i(T^\dagger - T) = T^\dagger T\,.$$

Expanding T in powers of g unitarity can be easily shown order-by-order. Let us introduce a complex effective coupling constant α defined as

$$\alpha = g + ig^2, \quad g = -\frac{i}{2}\left(\sqrt{1+4\,i\,\alpha}-1\right).$$

The T-matrix and the conjugated T-matrix expressed in terms of α read

$$T = -i\left[\exp\frac{1}{2}\left(\sqrt{1+4\,i\,\alpha}-1\right)-1\right] = \alpha - \frac{i\,\alpha^2}{2} - \frac{7\,\alpha^3}{6} + \cdots .$$
$$T^\dagger = i\left[\exp\frac{1}{2}\left(\sqrt{1-4\,i\,\alpha^*}-1\right)-1\right] = \alpha^* + \frac{i\,\alpha^{*2}}{2} - \frac{7\,\alpha^{*3}}{6} + \cdots .$$

From the above expressions we have

$$i(T^\dagger - T) = i\,(\alpha^* - \alpha) + \cdots . \tag{1}$$

and

$$T^\dagger T = \alpha^* \alpha + \cdots . \tag{2}$$

Comparing Eqs. (1) and (2) unitarity cannot be verified order-by-order in α (α^*). It is very unlikely that in QFT the solution of this problem becomes more feasible.

The above discussion does not apply to the electroweak theory, where $\cos\theta_W$ becomes complex due to the complex Z and W boson masses,[5] as long as the expansion parameter, the fine structure constant α, remains real. Note that although in high energy physics calculations the CMS with complex α has been used,[5,16] the introduction of complex α in electroweak theory in CMS is not necessary.

In an EFT of the Roper resonance interacting with pions and nucleons using CMS the undressed propagator of the Roper resonance has a complex pole[10]

$$iS_0(p) = \frac{i}{\not p - z}.$$

To see the implications of the pole on the first Riemann sheet consider πN scattering in the Roper resonance region. First diagram in Fig. 1 is the leading contribution. The third diagram cancels double pole of the second diagram on the second sheet. The pole on the first sheet is not canceled. Moreover, the double pole on the physical sheet remains. However this pole is far away from the applicability of perturbation

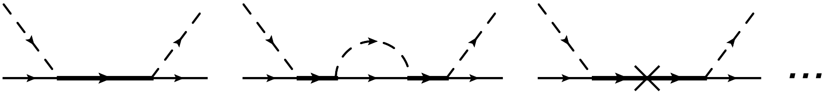

Fig. 1. Diagrams of πN scattering in Roper resonance region. Dashed, solid and bold solid lines correspond to pion, nucleon and Roper resonance, respectively. The cross stands for a counter-term.

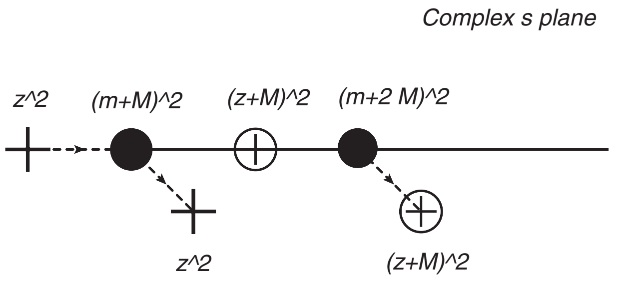

Fig. 2. Poles and branch points corresponding to the stable and unstable Roper resonance as described in the text. Solid line corresponds to the real axis.

theory. In that region one needs to re-sum self-energy insertions. The dressed propagator of the Roper resonance

$$iS_R(p) = \frac{i}{\not{p} - z_\chi - \Sigma_R(\not{p})}\,,$$

does not have poles on the physical sheet.

If the parameters of the theory are tuned so that the Roper resonance is stable then the πN scattering amplitude has a pole at $s = z^2$ on real axis in non-physical region and the corresponding branching point at $s = (z + M)^2$ is also real. By changing the parameters of the Lagrangian the stable Roper resonance becomes unstable ($Re(z) > m + M$). The corresponding pole and the branching point both move to the complex region as shown in Fig. 2.

3. EFT of Massive Vector Bosons

Consider an EFT of massive $SU(N)$ Yang-Mills fields interacting with fermions described by an effective Lagrangian[17]

$$\mathcal{L} = -\frac{1}{4}\, G^{a\mu\nu} G^a_{\mu\nu} + \frac{M^2}{2}\, B^{a\mu} B^a_\mu + \theta\, \tilde{G}^{a\alpha\beta} G^a_{\alpha\beta} + \sum_{q,i,j} \bar{\psi}^i_q \left(i \not{D}_{ij} - m_q\, \delta_{ij}\right) \psi^j_q + \mathcal{L}_1\,, \quad (3)$$

where B^a_μ are massive vector fields with the mass M, $G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g f^{abc} B^b_\mu B^c_\nu$, $D^\mu_{ij} = \delta_{ij}\,\partial^\mu - i g t^a_{ij} B^\mu_a$, $\not{D}_{ij} = \gamma_\mu D^\mu_{ij}$, $\tilde{G}^{a\alpha\beta} = \epsilon^{\mu\nu\alpha\beta} G^a_{\mu\nu}$, $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor, and t^a and f^{abc} are, respectively, the generators and the totally antisymmetric structure constants of the $SU(N)$ group. Summation runs over $q = 1, ..., n_f$ flavors of fermions. Lagrangian of Eq. (3) is invariant under local $SU(N)$ transformations except the mass term of vector bosons, which is only globally invariant. All possible Lorentz- and gauge-invariant terms (an infinite number of them) are contained in $\mathcal{L}_1$. It is assumed that contributions of renormalized couplings with negative mass dimensions in physical quantities are suppressed.

Symmetries of the effective action have been analyzed in Ref.[17] and the perturbative renormalizability of the considered EFT has been shown. Although the limit $M \to 0$ does not exist in perturbation theory, the analysis similar to the one of

Ref.[18] yields that this limit exists non-perturbatively and coincides with the effective theory of massless Yang-Mills fields.[17] This means that for the energies, much larger than the small mass of the vector boson and simultaneously much smaller than the large scale, standard massless Yang-Mills theory is reproduced.

Because of the non-zero mass of vector bosons, configurations with the slowly decaying ($\sim 1/r$ at infinity) asymptotics give vanishing contribution in the path integral. Therefore, as the term $\theta\,\epsilon^{\mu\nu\alpha\beta}G^a_{\mu\nu}G^a_{\alpha\beta}$ can be written as a total derivative, the absence of slowly decaying configurations guarantees that the θ-term will not have any effect on physical quantities. Thus EFT of massive Yang-Mills vector bosons is free of the CP problem without introducing any additional scalar field(s).[19,20]

On the other hand the $U(1)$ problem, which has been resolved by the presence of the instanton configurations,[21] seems to re-appear in the considered EFT. However, due to the stronger ultraviolet divergences of the considered EFT there appear fermion field-dependent contributions in the anomaly of the singlet axial current such that the parity doubling of the spectrum does not occur. Note that, the extension of the Adler-Bardeen theorem[22] to non-Abelian theories[23] does not apply here due to the non-renormalizability in the traditional sense. Note also that the path integral formalism[24–26] does not prove that there is only the standard one-loop contribution in the divergence of the singlet axial current.[27,28]

To consider the anomalous contributions to the divergence of the singlet axial current we use dimensional regularization. The divergence of the current reads:[29]

$$\partial_\mu J_5^\mu = -2\,i\,m_q\,\bar{\psi}_q\gamma_5\psi_q + \bar{\psi}_q\gamma_5\hat{\not{D}}\psi_q, \tag{4}$$

where the hat over the covariant derivative indicates that it vanishes in four space-time dimensions. The second term in (4) may lead to an anomaly from the loop contributions in the limit $n=4$, n being the space-time dimension.

We extract contributions to the divergence of the singlet axial current proportional to $1/M^2$ by calculating the matrix element $\langle 0|T(\bar{\psi}(x)\,\psi(y)\bar{\psi}_q\gamma_5\hat{\not{D}}\psi_q(0))|0\rangle$. One-loop diagrams contributing in this matrix element are shown in Fig. 3. Using dimensional regularization[30,29] we obtain:

$$\begin{aligned}&\int d^4x d^4y e^{ip'x-ipy}\langle 0|T(\bar{\psi}(x)\,\psi(y)\,\bar{\psi}_q\gamma_5\hat{\not{D}}\psi_q(0))|0\rangle\\&=\frac{g^2C_F}{4\,\pi^2}\,(\not{p}'-\not{p}\,)\,\gamma_5+\frac{g^2C_F}{16\pi^2M^2}\,\left(p'^2\not{p}-p^2\not{p}'\right)\gamma_5,\end{aligned} \tag{5}$$

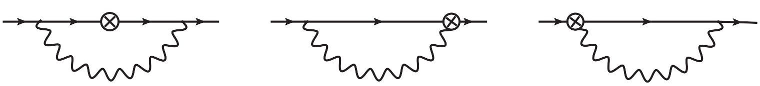

Fig. 3. One-loop diagrams giving contributions in the anomaly of the singlet axial current, proportional to $1/M^2$. Solid and wiggly lines correspond to fermions and vector bosons respectively, and the cross represents the divergence of the axial current.

where $\bar{\psi}_q \gamma_5 \hat{\not{D}} \psi_q(z)$ is a composite operator at point z and $C_F = (N^2-1)/2N$ is the Casimir operator of the $SU(N)$ group. The first term in Eq. (5) renormalizes the axial current and the second implies that there are additional contributions:

$$\partial_\mu J_5^\mu = \frac{n_f\, g^2}{32\,\pi^2}\, \epsilon^{\mu\nu\alpha\beta} G^a_{\mu\nu} G^a_{\alpha\beta} + \frac{g^2 C_F}{16\,\pi^2 M^2} \times \left(\partial^\mu \bar{\psi}\, \gamma^\mu \gamma_5 \partial^2 \psi + \partial^2 \bar{\psi}\, \gamma^\mu \gamma_5 \partial^\mu \psi \right) + \cdots\,, \tag{6}$$

where the first term is the standard fermion triangle loop contribution[27,28] and ellipses stand for terms of order $1/M^4$ as well as the terms which do not contribute in Eq. (5). Second term of the r.h.s. of Eq. (6) guarantees that the parity doubling of the spectrum does not occur. Indeed, although it is possible to define a new conserved axial current, the commutation relations of the corresponding axial charge with the fermion fields are not the ones generated by a symmetry operator.

To summarize, in the framework of the considered EFT of massive vector bosons interacting with fermions the $U(1)$ problem is absent and at the same time the CP violating term factors out from the dynamics.

Acknowledgments

J.G. was supported by the DFG (GE 2218/2-1) and the Georgian Shota Rustaveli National Science Foundation (grant 11/31).

References

1. J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999).
2. J. Gegelia, G. Japaridze and X. Q. Wang, J. Phys. G **29**, 2303 (2003).
3. T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D **68**, 056005 (2003).
4. R. G. Stuart, in Z^0 *Physics*, ed. J. Tran Thanh Van (Editions Frontiers, Gif-sur-Yvette, 1990), p.41.
5. A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. **B560**, 33 (1999).
6. A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Nucl. Phys. **B724**, 247 (2005).
7. S. Weinberg, "The Quantum Theory of Fields," *Cambridge, UK: Univ. Pr. (1995).*
8. D. Djukanovic, J. Gegelia and S. Scherer, Int. J. Mod. Phys. A **25**, 3603 (2010).
9. D. Djukanovic, J. Gegelia, A. Keller and S. Scherer, PoS CD **09**, 050 (2009)
10. D. Djukanovic, J. Gegelia and S. Scherer, Phys. Lett. B **690**, 123 (2010)
11. D. Djukanovic, J. Gegelia, A. Keller and S. Scherer, Phys. Lett. B **680**, 235 (2009)
12. T. Bauer, J. Gegelia, G. Japaridze and S. Scherer, Int. J. Mod. Phys. A **27**, 1250178 (2012)
13. T. Bauer, J. Gegelia and S. Scherer, Phys. Lett. B **715**, 234 (2012)
14. T. Bauer, D. Djukanovic, J. Gegelia, S. Scherer and L. Tiator, AIP Conf. Proc. **1432**, 269 (2012)
15. T. Bauer, J. Gegelia, G. Japaridze and S. Scherer, Int. J. Mod. Phys. A **27**, 1250178 (2012).
16. A. Denner and S. Dittmaier, Nucl. Phys. Proc. Suppl. **160**, 22 (2006).
17. J. Gegelia and G. Japaridze, Mod. Phys. Lett. A **27**, 1250128 (2012).
18. A. I. Vainshtein, I. B. Khriplovich, Yad. Fiz. **13**, 198-211 (1971).

19. R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).
20. R. D. Peccei and H. R. Quinn, Phys. Rev. D **16**, 1791 (1977).
21. G. 't Hooft, Phys. Rept. **142**, 357 (1986).
22. S. L. Adler and W. A. Bardeen, Phys. Rev. **182**, 1517 (1969).
23. G. Costa, J. Julve, T. Marinucci and M. Tonin, Nuovo Cim. A **38**, 373 (1977).
24. K. Fujikawa, Phys. Rev. Lett. **42**, 1195 (1979).
25. S. D. Joglekar and A. Misra, Phys. Rev. D **36**, 642 (1987).
26. S. D. Joglekar and A. Misra, Pramana **29**, 127 (1987).
27. S. L. Adler, Phys. Rev. **177**, 2426 (1969).
28. J. S. Bell and R. Jackiw, Nuovo Cim. A **60**, 47 (1969).
29. J. C. Collins, "*Renormalization*," *Cambridge, Uk: Univ. Pr. (1984).*
30. G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **44**, 189 (1972).