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# Configuration–Interaction Wave Functions and Transition Probabilities for N II

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## ABSTRACT

DEPARTMENT OF PHYSICS

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B.S. Umm Al-Qura University, 2012

### CONFIGURATION-INTERACTION WAVE FUNCTIONS AND TRANSITION

### PROBABILITIES FOR N II

Committee Chair: Swaraj S. Tayal, Ph.D.

Thesis dated December 2017

The energy levels, lifetimes, oscillator strengths, and transition probabilities of N II lines have been reported in this thesis. We have used the Hartree-Fock (HF) and Multiconfiguration Hartree-Fock (MCHF) methods in our calculations. The relativistic operators mass correction, one-body Darwin term, spin-orbit interaction, and spin-other-orbit have been included in the Breit-Pauli Hamiltonian in our calculations of atomic parameters of singly-ionized nitrogen. We considered 70 levels of the  $2s^2 2p^2$ ,  $2s^2 2p^3$ ,  $2s^2 2p 3p$ ,  $2s^2 2p 3s$ ,  $2s^2 2p 4p$ ,  $2s^2 2p 3d$ ,  $2s^2 2p 4s$ , and  $2s^2 2p 4d$  configurations of N II. Our results have been compared with other available calculations and measurements, and generally a good agreement is found.

CONFIGURATION-INTERACTION WAVE FUNCTIONS AND TRANSITION  
PROBABILITIES FOR N II

A THESIS

SUBMITTED TO THE FACULTY OF CLARK ATLANTA UNIVERSITY  
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR  
THE DEGREE OF MASTER OF SCIENCE

BY

KHULUD SAMNODI

DEPARTMENT OF PHYSICS

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## LIST OF ABBREVIATIONS

HF	Hartree-Fock
MCHF	Multiconfiguration Hartree-Fock
CSFs	Configuration State Functions
au	Atomic Units
CFF	Charlotte Froese Fischer
Ry	Rydberg
UV	Ultraviolet
CFG	Configuration
E1	Electric dipole
NASA	National Aeronautics and Space Administration
NIST	National Institute of Standards and Technology
$f_l$	Oscillator strengths in length formulation
$f_v$	Oscillator strengths in velocity formulation
$A_l$	Transition probabilities in length formulation
$A_v$	Transition probabilities in velocity formulation
$g_i$	Statistical weight of the initial state
$g_k$	Statistical weight of the final state

## CHAPTER I

### INTRODUCTION

Nitrogen is important because it is one of the elements important to life. However, unlike carbon, nitrogen is not a major part of rocks and minerals. This means that any organic nitrogen found in the soil of another planet likely would be the result of biological activity. Nitrogen was originally found in the atmosphere of Saturn's moon, Titan, and traces of it were found in the atmospheres of Venus and Mars, and a thick layer of liquid nitrogen covers Neptune's largest moon. Nitrogen is found both in a gaseous and liquid state in space, depending on where it is located. Outside of Earth's atmosphere, astronomers have found evidence of molecular nitrogen in the clouds of gas between the distant stars and galaxies. Nitrogen is formed through the CNO (Carbon-Nitrogen-Oxygen) cycle in the second generation of stars. When stars undergo fusion, nitrogen is a byproduct. Helium in stars fused to form heavier atoms, like nitrogen. This process is called nucleosynthesis.

A number of optical, ultraviolet, and extreme ultraviolet features of N II have been observed in the emission spectra of Titan's upper atmosphere, Saturn's inner magnetosphere, Sun, and other astrophysical objects. The N II intercombination or spin-forbidden and allowed lines are important in the density and temperature diagnostics of astrophysical plasmas. The transition probabilities for intercombination lines are sensitive to the choice of wave functions and adequate account of term mixing and correlation

effects are needed. In addition, transition probabilities of N II are needed to achieve good fits to high-resolution observations from space missions such as Cassini, the Hubble Space Telescope, and the Solar and Heliosphere Observatory. In this thesis, we studied observed spectral features due to N II ions [1-4].

Knowledge of the spectrum of ionized nitrogen is fundamental to the understanding of the Earth-ionosphere and comparative plasmas everywhere else in the solar system. While the N II spectrum is well understood we consider here some ultraviolet transition probabilities which appear to have anomalous values caused by strong mixing between overlapping configurations [5-6]. These various emission features of N II have been seen in the spectra of Titan atmosphere by Cassini Ultraviolet Imaging Spectrograph (UVIS). Titan's upper atmosphere is ionized by solar radiation and by electrons impinging from Saturn's magnetosphere. The Cassini Plasma Spectrometer (CAPS) observed N II in Saturn's inward magnetosphere. It is important to calculate nitrogen disappearance from Enceladus and Titan to comprehend the structure and the process of nitrogen ions creation in Saturn's magnetospheric plasma [7].

The N II wave functions exhibit large correlation effects and term dependence of the valence orbitals. We used orthogonal orbitals in the multi-configuration Hartree-Fock approach to calculate energy and oscillator strengths. The orthogonal spectroscopic and correlation orbitals are used for the accurate representation of N II states. Two separate sets of correlation orbitals ( $l = 0 - 4$ ) were generated for even and odd parity states. The mass correction and one-body Darwin term will shift the energy as a states as a whole energy but these will not split the state into the fine-structure levels. However, the spin-orbit interaction and spin-other-orbit interaction will split the term into the fine-structure

levels. The eigenvectors and eigenvalues are obtained by the diagonalization of the Breit-Pauli Hamiltonian.

In this thesis, we will present our calculation for the energy levels, lifetimes, line strengths, oscillator strengths, and transition probabilities for the electric dipole (E1) transitions. The results of present calculations have been compared with the data from NIST compilation, Tayal, Bell *et al*, and Froese Fischer. Professor Froese Fischer is a major contributor to the field of atomic computational atomic structure research [8]. The calculations presented in this thesis have been performed using the HF and MCHF computational methods. We have also used the semi-relativistic MCHF code based on the Breit-Pauli Hamiltonian. This approach uses the perturbation theory to include the relativistic corrections. The general agreement between our results and other computational results is satisfactory, except for some transitions where there are significant discrepancies. The discrepancies are especially larger for some weak semi-forbidden transitions with small oscillator strengths due to cancellation effects.

## CHAPTER II

### COMPUTATIONAL METHODS

To comprehend the atomic properties of N II we performed both LS-coupling and LSJ-coupling calculations. N II is a multielectron ion with six electrons, two electrons in the 1s core-shell and four electrons outside the core. Since we have more than one electron we should use an approximation approach to solve the Schrödinger's equation. In this chapter, we describe two coupling schemes of calculations in detail and provide an overview of the methods used [9-10].

#### A. Energies of States in LS-coupling

Any N-electron system can be defined by the wave function  $\psi(q_1, \dots, q_N)$  where  $q_i = (r_i, \sigma_i)$  and  $r_i$  are the space and  $\sigma_i$  are the spin coordinates of the electron labeled  $i$ . This wave function is a solution of the time-independent Schrödinger wave equation [11].

$$\mathcal{H}\psi(q_1, \dots, q_N) = E\psi(q_1, \dots, q_N), \quad (1)$$

where  $\mathcal{H}$  is the Hamiltonian operator of the atomic system, and E is the eigenvalue of the Hamiltonian operator. The eigenvalue E represents the total energy of the system. The eigenfunction is the atomic state wave function  $\psi(q_1, \dots, q_N)$ . The Hamiltonian  $\mathcal{H}$  for atomic system is given by

$$\mathcal{H} = \sum_{i=1}^N \left( -\frac{1}{2} \nabla_i^2 - \frac{Z}{r_i} \right) + \sum_{i>j}^N \frac{1}{r_{ij}}, \quad (2)$$

where  $Z$  is the nuclear charge of the atom,  $r_i$  is the distance of electron  $i$  from the nucleus, and  $r_{ij}$  is the distance between electrons  $i$  and  $j$ . The Hamiltonian is only correct if it is assumed that the relativistic effects can be ignored and the nucleus is a point charge with an infinite mass. The wave functions or eigenfunctions are given by

$$\psi(LS) = \sum_{i=1}^M a_i \phi_i(\alpha_i; LS), \quad (3)$$

where  $L$  and  $S$  are the total orbital and spin angular momenta,  $M$  is the number of configurations and  $\alpha_i$  specifies the angular momenta coupling scheme of the  $i^{\text{th}}$  configuration,  $a_i$  is the normalization constant, and  $\phi_i$  are the configuration state function. These wave functions are normalized

$$\int_{\mathbf{a}} |\psi(q_1, \dots, q_N)|^2 dq_1, \dots, dq_N \equiv \langle \psi | \psi \rangle = 1, \quad (4)$$

where the integration is over all space and spin co-ordinates respectively.

We obtain the approximate wave functions by replacing the full Hamiltonian  $\mathcal{H}$  with a separable Hamiltonian. The separable Hamiltonian is given by

$$\mathcal{H} \approx \mathcal{H}_0 = \sum_{i=1}^N \left( -\frac{1}{2} \nabla_i^2 - \frac{Z}{r_i} + v(r_i) \right), \quad (5)$$

where the potential  $v(r_i)$  is an approximation for the Coulomb repulsion among the electrons.

The separable Hamiltonian similar to the full Hamiltonian commutes with the total angular momentum operators  $L^2$ ,  $L_z$ ,  $S^2$ , and  $S_z$ . We may choose the wave functions of the separable Hamiltonian  $\mathcal{H}_0$  to be the wave functions of these operators as well.

$$\mathcal{H}_0\psi_0(q_1, \dots, q_N) = E_0\psi_0(q_1, \dots, q_N). \quad (6)$$

$E_0$  and  $\psi_0$  can be written as [11]

$$E_0 = \sum_{i=1}^N E_i, \quad (7)$$

and

$$\psi_0(q_1, \dots, q_N) = \prod_{i=1}^N \phi(\alpha_i; q_i). \quad (8)$$

The one-electron eigenfunction is given by

$$\phi(\alpha_i; q_i) = \frac{1}{r} P(nl; r) Y_{lm_l}(\theta, \varphi) X_{m_s}(\sigma), \quad (9)$$

where the radial eigenfunction  $P(nl; r)$  depends only on  $nl$  quantum numbers.

The Hamiltonian is stable with regard to permutations of the electron coordinates. Consequently, any permutation of the co-ordinates in the product function also leads to a wave function

$$\Phi(q_1, \dots, q_N) = \mathcal{A} \prod_{i=1}^N \phi(\alpha_i; q_i), \quad (10)$$

where  $\mathcal{A}$  is an antisymmetric function. This function can be expressed by the Slater determinant.

$$\Phi(q_1, \dots, q_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi(\alpha_1; q_1) & \dots & \phi(\alpha_1; q_N) \\ \vdots & \ddots & \vdots \\ \phi(\alpha_N; q_1) & \dots & \phi(\alpha_N; q_N) \end{vmatrix}. \quad (11)$$

In this equation, it is seen that the total wave function  $\Phi(q_1, \dots, q_N)$  is zero when two electrons have the same space and spin co-ordinates. According to the Pauli exclusion principle, there can be only one electron in each spin orbital. In the MCHF approach, the wave functions are approximated by a linear combination of orthonormal configuration state functions [11]

$$\Psi(\gamma LS) = \sum_{i=1}^M c_i \Phi(\gamma_i LS), \quad (12)$$

where

$$\sum_{i=1}^M c_i^2 = 1. \quad (13)$$

### B. Energies of States in LSJ-coupling

To estimate the relativistic impact in our N II system we use the Breit-Pauli Hamiltonian to correct our non-relativistic Hamiltonian. The Breit-Pauli Hamiltonian is given by [11]

$$\mathcal{H}_{BP} = \mathcal{H}_{NR} + \mathcal{H}_{RS} + \mathcal{H}_{FS}, \quad (14)$$

where  $\mathcal{H}_{NR}$  is the non-relativistic many-electron Hamiltonian.  $\mathcal{H}_{RS}$  is the relativistic shift operator and these commute with L and S and is given by



$$\mathcal{H}_{RS} = \mathcal{H}_{MC} + \mathcal{H}_{D1} + \mathcal{H}_{D2} + \mathcal{H}_{OO} + \mathcal{H}_{SSC}, \quad (15)$$

where  $\mathcal{H}_{MC}$  is the mass correction term

$$\mathcal{H}_{MC} = -\frac{\alpha^2}{8} \sum_{i=1}^N (\nabla_i^2) + \nabla_i^2, \quad (16)$$

$\mathcal{H}_{D1}$  and  $\mathcal{H}_{D2}$  are the one and two-body Darwin terms given by

$$\mathcal{H}_{D1} = -\frac{\alpha^2 Z}{8} \sum_{i=1}^N \nabla_i^2 \left( \frac{1}{r_i} \right), \quad (17)$$

and

$$\mathcal{H}_{D2} = \frac{\alpha^2}{4} \sum_{i<j}^N \nabla_i^2 \left( \frac{1}{r_{ij}} \right), \quad (18)$$

respectively.

Here  $\mathcal{H}_{OO}$  is the orbit-orbit term given by

$$\mathcal{H}_{OO} = -\frac{\alpha^2}{2} \sum_{i<j}^N \left[ \frac{p_i p_j}{r_{ij}} + \frac{r_{ij} (r_{ij} \cdot p_i) p_j}{r_{ij}^3} \right], \quad (19)$$

Finally,  $\mathcal{H}_{SSC}$  is the spin-spin contact term given by

$$\mathcal{H}_{SSC} = -\frac{8\pi\alpha^2}{3} \sum_{i<j}^N (s_i \cdot s_j) \delta(r_i \cdot r_j), \quad (20)$$

The fine-structure operator  $\mathcal{H}_{FS}$  describes the interactions between the spin and orbital angular momenta of the system. Also, the  $\mathcal{H}_{FS}$  does not commute with  $L$  and  $S$  but this operator only commutes with the total angular momentum operator  $J = L + S$  [11].

$$\mathcal{H}_{FS} = \mathcal{H}_{SO} + \mathcal{H}_{SOO} + \mathcal{H}_{SS}, \quad (21)$$

where  $\mathcal{H}_{SO}$  is the spin-orbit term given by

$$\mathcal{H}_{SO} = \frac{\alpha^2 Z}{2} \sum_{i=1}^N \frac{1}{r_i^3} l_i \cdot s_i, \quad (22)$$

$\mathcal{H}_{SOO}$  is the spin-other-orbit term given by

$$\mathcal{H}_{SOO} = -\frac{\alpha^2}{2} \sum_{i < j}^N \frac{r_{ij} \times p_i}{r_{ij}^3} (s_i + 2s_j), \quad (23)$$

$\mathcal{H}_{SS}$  is the spin-spin term given by

$$\mathcal{H}_{SS} = \alpha^2 \sum_{i < j}^N \frac{1}{r_{ij}^3} \left[ s_i \cdot s_j - 3 \frac{(s_i \cdot r_{ij})(s_j \cdot r_{ij})}{r_{ij}^2} \right]. \quad (24)$$

The Breit-Pauli wave functions are written as linear combination as follows

$$\Psi(\gamma J M_J) = \sum_{i=1}^M c_i \Phi(\gamma_i L_i S_i J M_J), \quad (25)$$

where each of M single-configuration functions  $\Phi$  is constructed from one-electron functions, and  $\gamma$  defines the coupling of angular momenta of the electrons. Also,  $c_i$  is mixing coefficient.

$$\Phi(\gamma_i L_i S_i J M_J) = \sum_{M_L M_S} \langle L M_L S M_S | L S J M_J \rangle \Phi(\gamma L M_L S M_S). \quad (26)$$

LSJ coupled CFGs are noted to be represented as  $\Phi(\gamma_i L_i S_i J M_j)$ . CFGs also have different LS terms that are contained within the above expansion as L and S are not good quantum numbers. The intermediate coupling is otherwise known as the wave equation that is given by [11].

$$Hc = Ec, \quad (27)$$

$$H_{ij} = \langle \gamma_i L_i S_i J M_j | \mathcal{H}_{BP} | \gamma_j L_j S_j J M_j \rangle, \quad (28)$$

The relativistic energy for many-electron system is given by

$$E = E_{NR} + E_{RS} + E_{FS}, \quad (29)$$

where  $E_{NR}$  is the non-relativistic energy given by

$$E_{NR} = \langle \gamma L S J M_j | \mathcal{H}_{NR} | \gamma L S J M_j \rangle, \quad (30)$$

here  $E_{RS}$  is the relativistic shift contribution and  $E_{FS}$  is the fine structure contributes to the relativistic energy correction.

$$E_{RS} = \langle \gamma L S J M_j | \mathcal{H}_{RS} | \gamma L S J M_j \rangle, \quad (31)$$

and

$$E_{FS} = \langle \gamma L S J M_j | \mathcal{H}_{FS} | \gamma L S J M_j \rangle, \quad (32)$$

The fine structure can be expressed as

$$E_{FS} = E_{SO} + E_{SOO} + E_{SS}, \quad (33)$$

where  $E_{SO}$ ,  $E_{SOO}$ , and  $E_{SS}$  are the energies corresponding to the spin-orbit, spin-other-orbit and spin-spin operators, respectively [11].

In our Breit-Pauli result, we neglect the spin-spin, orbit-orbit, and second Darwin interaction terms. The eigenfunctions obtained in equation (5) can be utilized to define the

length and velocity forms of the oscillator strengths and transition probabilities for transitions among the fine-structure levels. The oscillator strength can be referred as absorption or emission oscillator strength. The absorption oscillator strength is written as

$$f^{\pi k}(\gamma J, \gamma' J') = \frac{1}{\alpha} C_k [\alpha(E_{\gamma' J'} - E_{\gamma J})]^{2k-1} \frac{S^{\pi k}(\gamma J, \gamma' J')}{g_J}, \quad (34)$$

where the atom in the lower state absorbs a photon, and is excited to an upper state.

where  $g_{J'}$  is the statistical weight of the upper level given below

$$g_{J'} = 2J' + 1, \quad (35)$$

and

$$C_k = \frac{(2k+1)(k+1)}{k((2k+1)!!)^2}. \quad (36)$$

$S^{\pi k}(\gamma J, \gamma' J')$  is the line strength given by

$$S^{\pi k}(\gamma J, \gamma' J') = \sum_{M, M', q} \left| \langle \gamma J M | O_q^{\pi(k)} | \gamma' J' M' \rangle \right|^2, \quad (37)$$

where  $O_q^{\pi(k)}$  is a general transition operator of rank  $k$  and parity  $\pi$ .

For electric multipole transition ( $\pi = (-1)^k$ ) and magnetic multipole transition ( $\pi = (-1)^{k+1}$ ), the electric multipole transition operator is given below

$$E_q^{(k)} = \sum_{i=1}^N r^k(i) C_q^{(k)}(i), \quad (38)$$

The magnetic multipole form of the transition operator

$$M_q^{(k)} = \alpha \sqrt{k(2k+1)} \left[ \frac{1}{k+1} MA_q^{(k)} + \frac{1}{2} g_s MB_q^{(k)} \right], \quad (39)$$

where

$$MA_q^{(k)} = \sum_{i=1}^N r^{k-1}(i) [c^{(k-1)}(i) \times L^{(1)}(i)]_q^{(k)}, \quad (40)$$

and

$$MB_q^{(k)} = \sum_{i=1}^N r^{k-1}(i) [c^{(k-1)}(i) \times S^{(1)}(i)]_q^{(k)}, \quad (41)$$

To know how the transition occurs between an upper state and a lower state, we start with the transition integral

$$L_q^{\pi k}(\gamma JM, \gamma' J' M') = \langle \gamma JM | O_q^{\pi(k)} | \gamma' J' M' \rangle. \quad (42)$$

The component strength is given by

$$S^{\pi k}(\gamma JM, \gamma' J' M') = \sum_q |L_q^{\pi k}(\gamma JM, \gamma' J' M')|^2. \quad (43)$$

The transition probability from the upper level to the lower level is given by

$$A^{\pi k}(\gamma J, \gamma' J') = 2C_k [\alpha(E_{\gamma' J'} - E_{\gamma J})]^{2k+1} \frac{S^{\pi k}(\gamma J, \gamma' J')}{g_{J'}}. \quad (44)$$

The lifetime,  $\tau_{\gamma'J'}$  is inverse of the sum of transition probabilities over the multipole transitions to all the lower energy levels [11-13]

$$\tau_{\gamma'J'} = \frac{1}{\sum_{\pi k, \gamma J} A^{\pi k}(\gamma J, \gamma' J')} \quad (45)$$

## CHAPTER III

### RESULTS AND DISCUSSION

The results from the present LS-coupling and LSJ-coupling calculations are presented in this chapter. We show the results for the energy levels, lifetimes, oscillator strengths and transition probabilities for E1 transitions.

#### A. Wave Functions

The N II spectroscopic wave functions generated in our calculations are exhibited in Figures 1-3. Each figure demonstrates the wave function as a function of the radius. In Figure 1, we present 1s, 2s, 3s, and 4s radial wave functions which are obtained on the  $2s^2 2p^2 \ ^3P$  ground state. In Figure 2, we show 2p, 3p, and 4p radial wave functions, and Figure 3 shows 3d and 4d radial wave functions of the same orbital angular momentum. It can be seen from these figures that the wave functions follow the rule for the number of nodes given by  $n - l - 1$ .

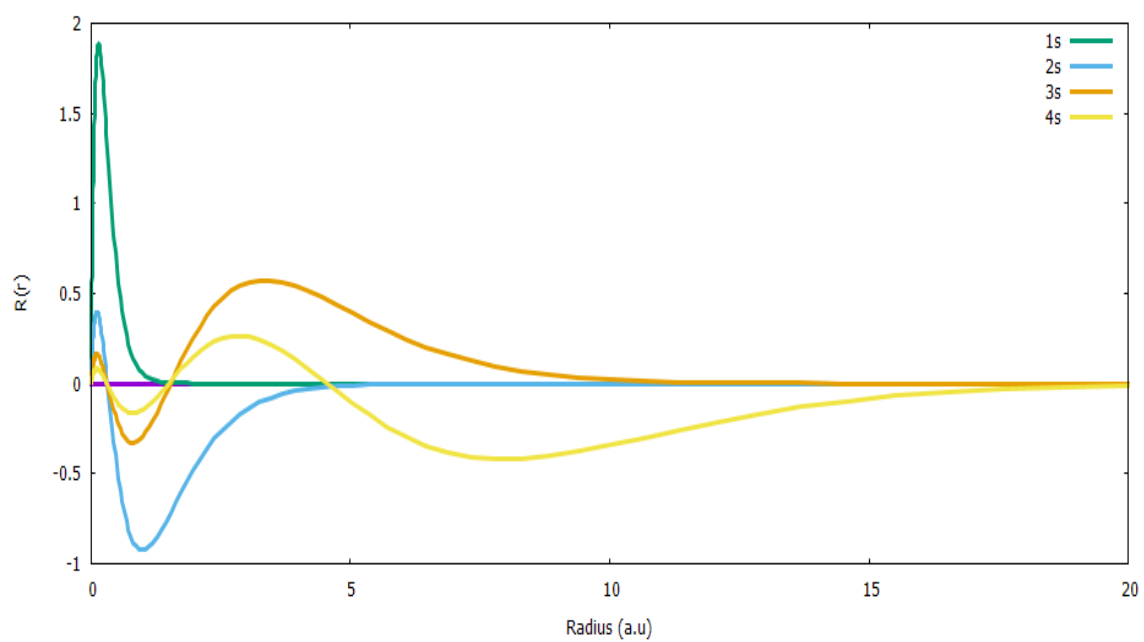


FIG 1. 1s, 2s, 3s, and 4s radial wave functions.

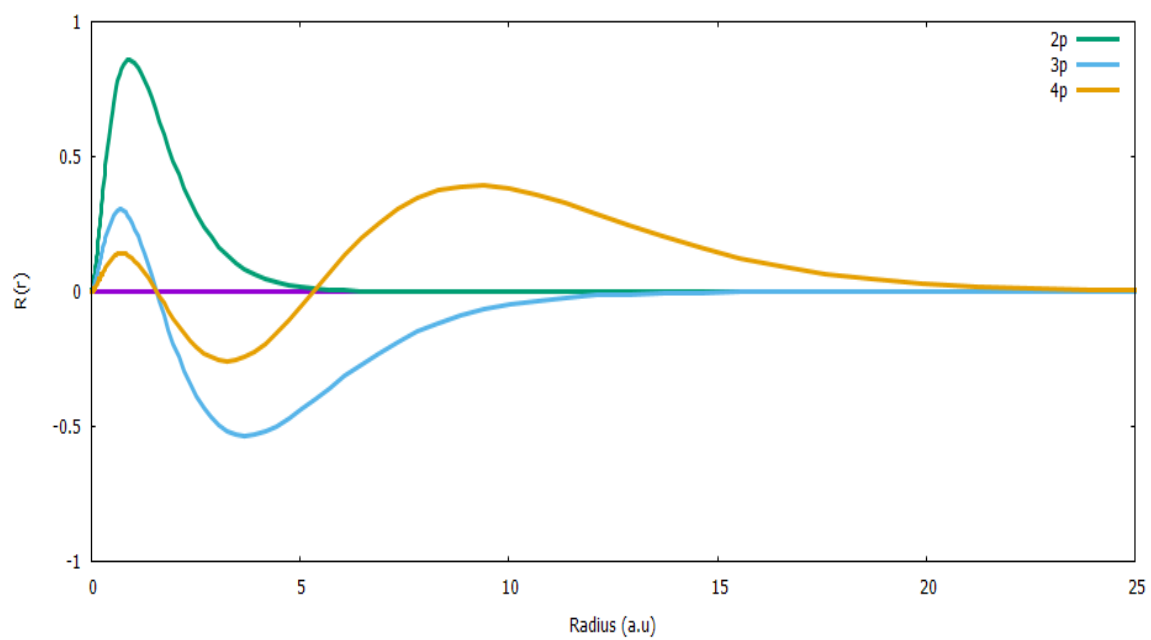


FIG 2. 2p, 3p, and 4p radial wave functions.



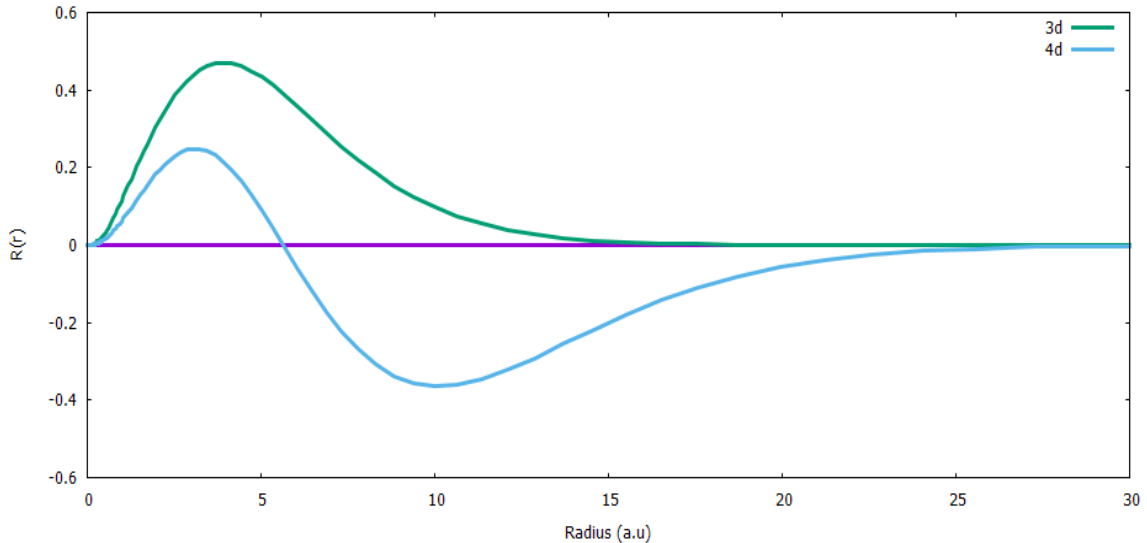


FIG 3. 3d and 4d radial wave functions.

### B. Excitation Energies

The calculated excitation energies of fine-structure levels relative to the ground level are presented in Table 1, where we compare these energies with the calculated results of Tayal (2011) and NIST compilation (<http://physics.nist.gov>). We note that some of the energy states calculated in the present work were not given in Tayal calculations. These states are  $2s^22p4d\ ^3F$ ,  $2s^22p4d\ ^1D$ ,  $2s^22p4d\ ^3D$ ,  $2s^22p4d\ ^3P$ ,  $2s^22p4d\ ^1F$ , and  $2s^22p4d\ ^1P$ . The wave functions in our calculations are predominately focused on accurate representation of the  $2s^22p^2\ ^3P$  ground state and the excited  $2s2p^3\ ^5S^o$ ,  $2s^22p3s\ ^3P^o$ ,  $^1P^o$ ,  $2s^22p3p\ ^1P$ ,  $^3D$ ,  $^3S$ ,  $^1D$ , and  $^1S$  states. We included all energy states up to the  $2s^22p4d\ ^1P^o$ . The states of the  $2s^22p3d$  configuration are very well represented. The general agreement between NIST data and our results is satisfactory, with average deviation of around 0.002 Ry for most of the states. However, the deviation of around

0.01 Ry is noted for the  $2s2p^3\ ^5S^o$ ,  $2s^22p3p\ ^1S$ ,  $2s^22p4s\ ^3P^o$ ,  $2s^22p4p\ ^3P$ ,  $^3S$ ,  $^1D$ ,  $2s^22p4d\ ^3F^o$ ,  $^1D^o$ ,  $^3D^o$ ,  $^3P^o$ ,  $^1F^o$ , and  $2s^22p^23s\ ^3P$  states. The  $2s2p^3\ ^1P$  state deviates around 0.062 Ry from NIST compilation. The presently calculated energies generally agree by about 0.5% with energies calculated by Tayal. We also found that the largest discrepancy is for the  $2s2p^3\ ^1P$  state where the present calculated value is higher than the result of Tayal by about 2%. We found discrepancy for the  $2s^22p3p\ ^1D$  and  $2s^22p4s\ ^3P^o$  states where the present calculations are lower than the results of Tayal by about 0.06%.

### C. Lifetimes

We have calculated transition probabilities of all possible E1 transitions between the terms of the ground  $2s^22p^2$  and the excited  $2s2p^3$ ,  $2s^22p3s$ ,  $2s^22p3p$ ,  $2s^22p3d$ ,  $2s^22p4s$ ,  $2s^22p4p$ , and  $2s^22p4d$  configurations. We present lifetimes in Table 2 where the comparison has been made with Tayal (2011) and the results of Froese Fischer (2004). The present lifetime for the  $2s^22p^3\ ^1D^o$  state is in excellent agreement with the calculation of Tayal. In general, the present lifetimes are about 12% of the previous calculation for most of the levels [14].

**Table 1.** Comparison of Energies (in Ry) of the N II States with NIST data and Tayal Calculation.

Index	CFS	LS	J	Present	NIST <sup>a</sup>	Tayal <sup>b</sup>
1	$2s^22p^2$	$^3p$	0	0.0000	0.0000	0.0000
2			1	0.0004	0.0004	0.0004
3			2	0.0012	0.0011	0.0011
4	$2s^22p^2$	$^1D$	2	0.1516	0.1395	0.1499
5	$2s^22p^2$	$^1S$	0	0.3434	0.2978	0.3027
6	$2s^22p^3$	$^5S^o$	2	0.4336	0.4263	0.4145
7	$2s^22p^3$	$^3D^o$	3	0.8626	0.8405	0.8575
8			2	0.8626	0.8406	0.8576
9			1	0.8625	0.8406	0.8577
10	$2s^22p^3$	$^3P^o$	1	1.0417	0.9952	1.0195
11			2	1.0418	0.9952	1.0195
12			0	1.0417	0.9953	1.0195
13	$2s^22p^3$	$^1D^o$	2	1.3434	1.3139	1.3520
14	$2s^22p3s$	$^3P^o$	0	1.3532	1.3569	1.3583
15			1	1.3536	1.3572	1.3587
16			2	1.3548	1.3584	1.3598
17	$2s^22p3s$	$^1P^o$	1	1.3594	1.3595	1.3653
18	$2s^22p^3$	$^3S^o$	1	1.4423	1.4136	1.4301
19	$2s^22p3p$	$^1P$	1	1.4922	1.5000	1.5075
20	$2s^22p3p$	$^3D$	1	1.5106	1.5174	1.5197

21			2	1.5112	1.5180	1.5215
22			3	1.5120	1.5188	1.5220
23	$2s^22p^3$	$^1P^o$	1	1.5816	1.5196	1.5451
24	$2s^22p3p$	$^3S$	1	1.5314	1.5390	1.5467
25	$2s^22p3p$	$^3P$	0	1.5434	1.5543	1.5562
26			1	1.5516	1.5546	1.5565
27			2	1.5520	1.5552	1.5570
28	$2s^22p3p$	$^1D$	2	1.5897	1.5875	1.5882
29	$2s^22p3p$	$^1S$	0	1.6380	1.6245	1.6226
30	$2s^22p3d$	$^3F^o$	2	1.6982	1.6996	1.7039
31			3	1.6986	1.7001	1.7045
32			4	1.6992	1.7009	1.7052
33	$2s^22p3d$	$^1D^o$	2	1.7044	1.7049	1.7088
34	$2s^22p3d$	$^3D^o$	1	1.7068	1.7080	1.7116
35			2	1.7072	1.7082	1.7118
36			3	1.7072	1.7085	1.7120
37	$2s^22p3d$	$^3P^o$	2	1.7194	1.7210	1.7262
38			1	1.7198	1.7214	1.7266
39			0	1.7200	1.7217	1.7269
40	$2s^22p3d$	$^1F^o$	3	1.7266	1.7253	1.7299
41	$2s^22p3d$	$^1P^o$	1	1.7347	1.7325	1.7677
42	$2s^22p4s$	$^3P^o$	0	1.7851	1.7910	1.7871

43			1	1.7856	1.7914	1.7875
44			2	1.7866	1.7925	1.7885
45	$2s^2 2p 4s$	$^1P^o$	1	1.7994	1.8030	1.8277
46	$2s^2 2p 4p$	$^1P$	1	1.8364	1.8423	1.8405
47	$2s^2 2p 4p$	$^3D$	1	1.8402	1.8472	1.8470
48			2	1.8405	1.8477	1.8475
49			3	1.8414	1.8486	1.8483
50	$2s^2 2p 4p$	$^3P$	0	1.8424	1.8513	1.8516
51			1	1.8428	1.8515	1.8518
52			2	1.8433	1.8522	1.8524
53	$2s^2 2p 4p$	$^3S$	1	1.8460	1.8547	1.8562
54	$2s^2 2p 4p$	$^1D$	2	1.8538	1.8712	1.8694
55	$2s 2p^2$	$^5P$	1	1.8561	1.8735	1.8761
56			2	1.8566	1.8740	1.8766
57			3	1.8572	1.8747	1.8773
58	$2s^2 2p 4p$	$^1S$	0	1.8865	1.8850	1.8801
59	$2s^2 2p 4d$	$^3F^o$	2	1.9052	1.9106	.....
60			3	.....	1.9112	.....
61			4	1.9064	1.9120	.....
62	$2s^2 2p 4d$	$^1D^o$	2	1.9074	1.9129	.....
63	$2s^2 2p 4d$	$^3D^o$	1	1.9099	1.9158	.....
64			2	1.9104	1.9160	.....

65			3	1.9106	1.9164	.....
66	2s <sup>2</sup> 2p4d	<sup>3</sup> P <sup>o</sup>	2	1.9144	1.9200	.....
67			1	1.9148	1.9205	.....
68			0	1.9149	1.9207	.....
69	2s <sup>2</sup> 2p4d	<sup>1</sup> F <sup>o</sup>	3	1.9196	1.9237	.....
70	2s <sup>2</sup> 2p4d	<sup>1</sup> P <sup>o</sup>	1	1.9238	1.9258	.....

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<sup>a</sup> A. Kramida, Yu. Ralchenko, J. Reader, and NIST ASD Team (2015). NIST Atomic Spectra Database, 5.3, [Online]. Available: <http://physics.nist.gov/asd>. National Institute of Standards and Technology, Gaithersburg, MD.

<sup>b</sup> S. S. Tayal (2011).

**Table 2.** Comparison of Lifetimes (in ns) of N II States with other Calculations.

Index	CFS	LS	J	Present	CFF <sup>a</sup>	Tayal <sup>b</sup>
1	2s <sup>2</sup> 2p <sup>2</sup>	<sup>3</sup> p	0	.....	.....	.....
2			1	.....	4.80E+05	.....
3			2	.....	1.34E+05	.....
4	2s <sup>2</sup> 2p <sup>2</sup>	<sup>1</sup> D	2	.....	2.56E+02	.....
5	2s <sup>2</sup> 2p <sup>2</sup>	<sup>1</sup> S	0	.....	8.56E-01	.....
6	2s <sup>2</sup> 2p <sup>3</sup>	<sup>5</sup> S <sup>o</sup>	2	7.08E-03	5.61E-03	6.14E-03
7	2s <sup>2</sup> 2p <sup>3</sup>	<sup>3</sup> D <sup>o</sup>	3	2.53E-09	2.68E-09	2.62E-09
8			2	2.52E-09	2.67E-09	2.61E-09
9			1	2.52E-09	2.67E-09	2.61E-09
10	2s <sup>2</sup> 2p <sup>3</sup>	<sup>3</sup> P <sup>o</sup>	1	6.70E-10	7.86E-10	7.53E-10
11			2	6.71E-10	7.88E-10	7.52E-10
12			0	6.69E-10	7.85E-10	7.51E-10
13	2s <sup>2</sup> 2p <sup>3</sup>	<sup>1</sup> D <sup>o</sup>	2	3.10E-10	3.18E-10	3.11E-10
14	2s <sup>2</sup> 2p3s	<sup>3</sup> P <sup>o</sup>	0	9.52E-10	8.81E-10	9.24E-10
15			1	9.24E-10	7.07E-10	8.72E-10
16			2	9.50E-10	8.80E-10	9.22E-10
17	2s <sup>2</sup> 2p3s	<sup>1</sup> P <sup>o</sup>	1	2.63E-10	2.61E-10	2.58E-10
18	2s <sup>2</sup> 2p <sup>3</sup>	<sup>3</sup> S <sup>o</sup>	1	8.60E-11	9.35E-11	8.96E-11
19	2s <sup>2</sup> 2p3p	<sup>1</sup> P	1	3.30E-08	3.08E-08	2.74E-08
20	2s <sup>2</sup> 2p3p	<sup>3</sup> D	1	1.23E-08	1.27E-08	1.18E-08

21			2	1.23E-08	1.27E-08	1.18E-08
22			3	1.23E-08	1.26E-08	1.18E-08
23	$2s^2 2p^3$	$^1P^o$	1	1.83E-10	2.21E-10	2.03E-10
24	$2s^2 2p^3 p$	$^3S$	1	7.00E-09	6.87E-09	6.66E-09
25	$2s^2 2p^3 p$	$^3P$	0	5.71E-09	6.14E-09	5.81E-09
26			1	5.70E-09	6.13E-09	5.81E-09
27			2	5.69E-09	6.13E-09	5.80E-09
28	$2s^2 2p^3 p$	$^1D$	2	6.71E-09	7.33E-09	6.89E-09
29	$2s^2 2p^3 p$	$^1S$	0	4.16E-09	4.72E-09	4.65E-09
30	$2s^2 2p^3 d$	$^3F^o$	2	6.81E-09	6.81E-09	4.16E-09
31			3	7.25E-09	7.59E-09	6.39E-09
32			4	8.08E-09	8.75E-09	8.39E-09
33	$2s^2 2p^3 d$	$^1D^o$	2	3.09E-10	3.46E-10	3.49E-10
34	$2s^2 2p^3 d$	$^3D^o$	1	2.13E-10	2.30E-10	2.34E-10
35			2	2.14E-10	2.31E-10	2.36E-10
36			3	2.13E-10	2.30E-10	2.36E-10
37	$2s^2 2p^3 d$	$^3P^o$	2	3.49E-10	3.90E-10	3.91E-10
38			1	3.49E-10	3.90E-10	3.91E-10
39			0	3.49E-10	3.90E-10	3.91E-10
40	$2s^2 2p^3 d$	$^1F^o$	3	2.31E-10	2.57E-10	2.73E-10
41	$2s^2 2p^3 d$	$^1P^o$	1	3.29E-10	3.72E-10	3.99E-10
42	$2s^2 2p^4 s$	$^3P^o$	0	2.38E-09	.....	.....



43			1	2.36E-09	.....	.....
44			2	2.36E-09	.....	.....
45	$2s^22p4s$	$^1P^o$	1	1.05E-09	.....	.....
46	$2s^22p4p$	$^1P$	1	1.07E-08	.....	.....
47	$2s^22p4p$	$^3D$	1	1.20E-08	.....	.....
48			2	1.19E-08	.....	.....
49			3	1.26E-08	.....	.....
50	$2s^22p4p$	$^3P$	0	3.32E-09	.....	.....
51			1	3.42E-09	.....	.....
52			2	3.37E-09	.....	.....
53	$2s^22p4p$	$^3S$	1	8.80E-09	.....	.....
54	$2s2p^2$	$^5P$	1	9.39E-10	.....	.....
55			2	9.38E-10	.....	.....
56			3	9.36E-10	.....	.....
57	$2s^22p4p$	$^1D$	2	1.25E-08	.....	.....
58	$2s^22p4p$	$^1S$	0	7.05E-09	.....	.....
59	$2s^22p4d$	$^3F^o$	2	9.34E-09	.....	.....
60			3	1.41E-08	.....	.....
61			4	2.27E-08	.....	.....
62	$2s^22p4d$	$^1D^o$	2	7.78E-10	.....	.....
63	$2s^22p4d$	$^3D^o$	1	4.27E-10	.....	.....
64			2	4.33E-10	.....	.....

65			3	4.30E-10	.....	.....
66	$2s^2 2p 4d$	$^3P^o$	2	6.71E-10	.....	.....
67			1	6.73E-10	.....	.....
68			0	6.76E-10	.....	.....
69	$2s^2 2p 4d$	$^1F^o$	3	3.59E-10	.....	.....
70	$2s^2 2p 4d$	$^1P^o$	1	4.41E-10	.....	.....

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<sup>a</sup> Froese Fischer, G. Tachiev, *Atomic Data and Nuclear Data Tables*, 87, 1-184 (2004).

<sup>b</sup> Tayal, S. S. 2011, *Phys. Rev. A*, 83, 012515.

#### D. Oscillator Strengths and Transition Probabilities

The length values of oscillator strengths and transition probabilities for dipole-allowed (E1) transitions between the fine-structure levels are shown in Table 3. The indices of initial and final levels of transitions are listed in Table 1. Our calculations are compared with the results of Froese Fischer (2004). In Figure 4, the ratio of the present velocity and length forms of the oscillator strength is displayed as a function of the present length form of oscillator strength for some dipole-allowed transitions. The data in this figure are found to lie between the two lines which represent a 20% deviation. In Figure 5, the ratio of oscillator strength from the calculation of Froese Fischer in length and the present length form of the oscillator strength is shown as a function of the present length form of oscillator strength for some dipole E1 transitions. Transition between the  $2s^2 2p^2 \ ^3P$  and  $2s^2 2p 3s \ ^3P^o$  levels have been chosen. The two calculations agree to within 20% for the oscillator strength of stronger transitions [15-18].

**Table 3.** Comparison of Length Oscillator Strengths ( $f_l$ ) and Transition Probabilities ( $A_l$ ) for E1 Transition with CFF Results

Transition				Present		CFF <sup>a</sup>	
Initial Level	Final Level	$g_i$	$g_k$	$f_l$	$A_l$	$f_l$	$A_l$
$2s^22p^2$ $^3P$	$2s2p^3$ $^3D$	1	3	1.11E-01	2.21E+08	1.10E-01	2.09E+08
$2s^22p^2$ $^3P$	$2s2p^3$ $^3D$	3	3	2.74E-02	1.63E+08	2.72E-02	1.54E+08
$2s^22p^2$ $^3P$	$2s2p^3$ $^3D$	3	5	8.33E-02	2.98E+08	8.29E-02	2.82E+08
$2s^22p^2$ $^3P$	$2s2p^3$ $^3P$	1	3	1.70E-01	4.96E+08	1.59E-01	4.22E+08
$2s^22p^2$ $^3P$	$2s2p^3$ $^3P$	3	3	4.32E-02	3.76E+08	4.04E-02	3.21E+08
$2s^22p^2$ $^3P$	$2s2p^3$ $^3P$	3	5	7.06E-02	3.69E+08	6.58E-02	3.13E+08
$2s^22p^2$ $^3P$	$2s^22p3s$ $^3P$	1	3	7.04E-02	3.45E+08	6.89E-02	3.40E+08
$2s^22p^2$ $^3P$	$2s^22p3s$ $^3P$	3	3	1.75E-02	2.58E+08	1.71E-02	2.53E+08
$2s^22p^2$ $^3P$	$2s^22p3s$ $^3P$	3	5	2.97E-02	2.63E+08	3.19E-02	2.84E+08
$2s^22p^2$ $^3P$	$2s^22p3s$ $^1P$	1	3	8.79E-04	4.35E+06	7.67E-03	3.79E+07
$2s^22p^2$ $^3P$	$2s^22p3s$ $^1P$	3	3	2.46E-04	3.65E+06	2.03E-03	3.01E+07
$2s^22p^2$ $^3P$	$2s2p^3$ $^3S$	1	3	2.31E-01	1.29E+09	2.22E-01	1.18E+09
$2s^22p^2$ $^3P$	$2s2p^3$ $^1P$	1	3	2.05E-06	1.37E+04	1.65E-06	1.02E+04
$2s^22p^2$ $^3P$	$2s^22p3d$ $^3D$	1	3	3.39E-01	2.64E+09	3.14E-01	2.46E+09

$2s^2 2p^2 \ ^3P$	$2s^2 2p 3d \ ^3D$	3	3	7.88E-02	1.84E+09	7.26E-02	1.70E+09
$2s^2 2p^2 \ ^3P$	$2s^2 2p 3d \ ^3P$	1	3	1.03E-01	8.16E+08	9.03E-02	7.17E+08
$2s^2 2p^2 \ ^3P$	$2s^2 2p 3d \ ^3P$	3	3	3.19E-02	7.57E+08	2.88E-02	6.85E+08
$2s^2 2p^2 \ ^3P$	$2s^2 2p 3d \ ^1P$	1	3	2.77E-04	2.23E+06	4.89E-04	3.93E+06
$2s^2 2p^2 \ ^3P$	$2s^2 2p 3d \ ^1P$	3	3	3.60E-06	8.69E+04	3.62E-06	8.73E+04
$2s^2 2p^2 \ ^1S$	$2s 2p^3 \ ^3D$	1	3	2.75E-07	1.98E+02	4.58E-07	3.61E+02
$2s^2 2p^2 \ ^1S$	$2s 2p^3 \ ^3P$	1	3	1.07E-06	1.39E+03	1.58E-06	2.05E+03
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3s \ ^3P$	1	3	3.45E-04	9.45E+05	8.82E-04	2.65E+06
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3s \ ^1P$	1	3	3.08E-02	8.52E+07	8.49E-03	2.56E+04
$2s^2 2p^2 \ ^1S$	$2s 2p^3 \ ^3S$	1	3	2.51E-06	8.13E+03	5.55E-06	1.85E+04
$2s^2 2p^2 \ ^1S$	$2s 2p^3 \ ^1P$	1	3	2.49E-01	1.02E+09	3.36E-01	1.34E+09
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3d \ ^3D$	1	3	2.04E-04	1.01E+06	2.67E-04	1.42E+06
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3d \ ^3P$	1	3	2.11E-04	1.07E+06	4.24E-04	2.30E+06
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3d \ ^1P$	1	3	5.60E-01	2.90E+09	4.68E-01	2.58E+09
$2s^2 2p 3p \ ^3P$	$2s^2 2p 3d \ ^3P$	1	3	1.16E-01	8.86E+06	1.31E-01	9.82E+06
$2s^2 2p 3p \ ^3P$	$2s^2 2p 3d \ ^1P$	1	3	1.54E-04	1.39E+04	4.56E-04	3.87E+04
$2s^2 2p 3p \ ^3P$	$2s^2 2p 3d \ ^3D$	1	3	4.92E-01	3.18E+07	5.04E-01	3.18E+07

$2s^2 2p^3 p$	$^1S$	$2s^2 2p^3 d$	$^3D$	1	3	9.84E-05	1.24E+03	1.64E-04	3.07E+03
$2s^2 2p^3 p$	$^1S$	$2s^2 2p^3 d$	$^3P$	1	3	1.53E-04	2.76+03	4.11E-04	1.03E+04
$2s^2 2p^3 p$	$^1S$	$2s^2 2p^3 d$	$^1P$	1	3	6.50E-01	1.63E+07	6.11E-01	1.90E+07
$2s 2p^3$	$^3P$	$2s^2 2p^3 p$	$^1P$	1	3	7.60E-06	4.13E+03	5.95E-06	4.05E+03
$2s 2p^3$	$^3P$	$2s^2 2p^3 p$	$^1S$	3	1	1.29E-08	1.11E+02	1.04E-08	9.98E+01
$2s^2 2p^3 s$	$^3P$	$2s^2 2p^3 p$	$^1P$	1	3	1.19E-04	6.18E+03	1.48E-04	8.14E+03
$2s^2 2p^3 s$	$^3P$	$2s^2 2p^3 p$	$^1S$	3	1	1.27E-03	2.48E+06	1.10E-02	1.90E+07

<sup>a</sup>Froese Fischer, G. Tachiev, Atomic Data and Nuclear Data Tables, 87, 1-184 (2004).

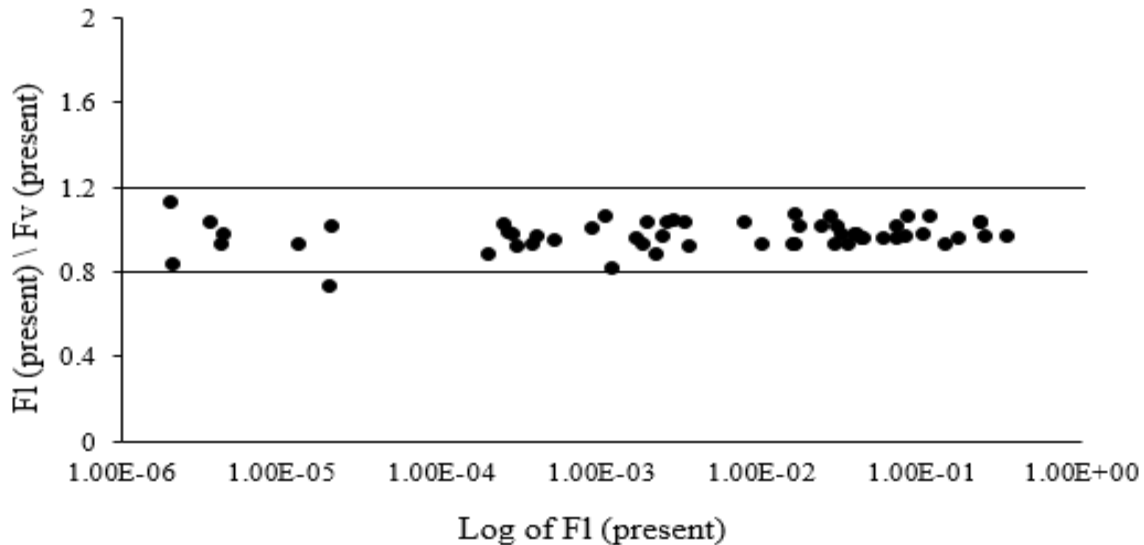


FIG 4. The ratio between length and velocity forms of the present oscillator strengths as a function of length form of present oscillator strengths for some dipole E1 transitions has been shown. The solid lines represent a 20% deviation.

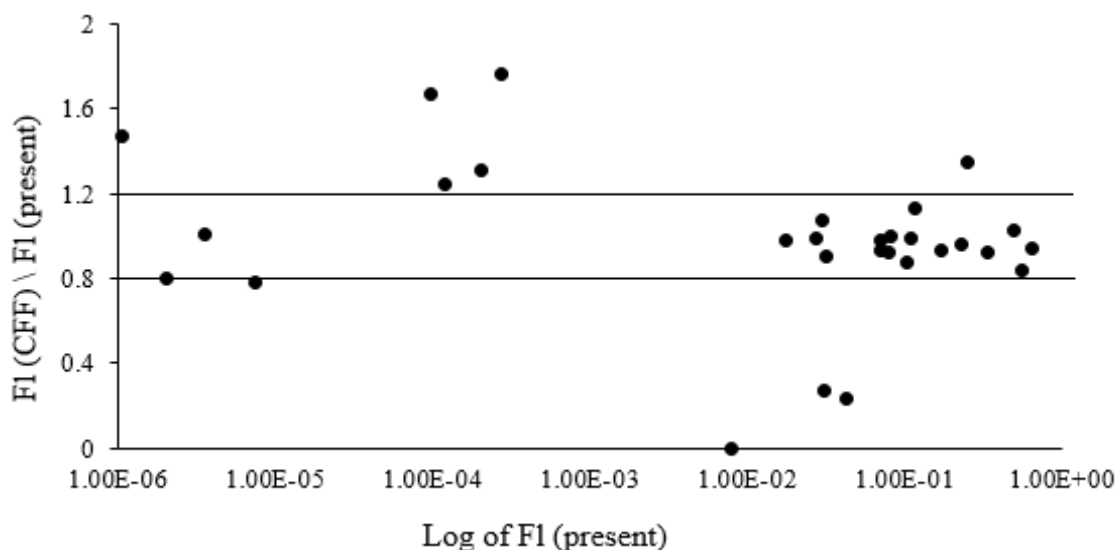


FIG 5. The ratio between length of Froese Fischer and the present oscillator strengths as a function of length form of the present oscillator strengths for some dipole E1 transitions has been shown. The solid lines represent a 20% deviation.

We have calculated oscillator strengths for E1 dipole-allowed transitions between the terms of the ground  $2s^22p^2$  and the excited  $2s2p^3$ ,  $2s^22p3s$ , and  $2s^22p3p$  configurations. The length and velocity forms of these calculations are presented in Table 4, and we have compared present results with those from the calculations of Bell *et al* (1995). The agreement is generally reasonably good. The ratio between the length of Bell *et al* and the present ratios of oscillator strengths in length and velocity formulations have been plotted in Figure 6 as a function of length form of the present oscillator strength for some dipole E1 transitions. The solid lines represent a 20% deviation. The agreement between the length of Bell *et al* and the present length values of oscillator strengths are within 20% for most of the transitions arising from the ground  $2s^22p^2$  configuration.

**Table 4.** Comparison of Length ( $f_l$ ) and Velocity ( $f_v$ ) Oscillator Strengths for E1 Transition with Bell *et al* Results

Transition				Present		Bell <i>et al</i> <sup>a</sup>	
Initial Level	Final Level	$g_i$	$g_k$	$f_l$	$f_v$	$f_l$	$f_v$
$2s^22p^2$ $^3P$	$2s2p^3$ $^3D$	1	3	1.11E-01	1.18E-01	1.13E-01	1.28E-01
$2s^22p^2$ $^3P$	$2s2p^3$ $^3D$	3	3	2.74E-02	2.93E-02	2.80E-02	3.15E-02
$2s^22p^2$ $^3P$	$2s2p^3$ $^3D$	3	5	8.33E-02	8.92E-02	8.52E-02	2.61E-02
$2s^22p^2$ $^3P$	$2s2p^3$ $^3P$	1	3	1.70E-01	1.63E-01	1.63E-01	1.77E-01
$2s^22p^2$ $^3P$	$2s2p^3$ $^3P$	3	3	4.32E-02	4.14E-02	4.13E-02	4.51E-02
$2s^22p^2$ $^3P$	$2s2p^3$ $^3P$	3	5	7.06E-02	6.78E-02	6.74E-02	7.36E-02
$2s^22p^2$ $^3P$	$2s^22p3s$ $^3P$	1	3	7.04E-02	7.14E-02	6.94E-02	7.19E-02
$2s^22p^2$ $^3P$	$2s^22p3s$ $^3P$	3	3	1.75E-02	1.78E-02	1.72E-02	1.79E-02
$2s^22p^2$ $^3P$	$2s^22p3s$ $^3P$	3	5	2.97E-02	3.02E-02	3.05E-02	3.16E-02
$2s^22p^2$ $^3P$	$2s^22p3s$ $^1P$	1	3	8.79E-04	8.88E-04	3.70E-03	3.90E-03
$2s^22p^2$ $^3P$	$2s^22p3s$ $^1P$	3	3	2.46E-04	2.52E-04	1.00E-03	1.10E-03
$2s^22p^2$ $^3P$	$2s2p^3$ $^3S$	1	3	2.31E-01	2.40E-01	2.25E-01	2.43E-01
$2s^22p^2$ $^3P$	$2s2p^3$ $^1P$	1	3	2.05E-06	2.32E-06	.....	.....

$2s^2 2p^2 \ ^3P$	$2s^2 2p^3 3d \ ^3D$	3	3	7.88E-02	7.67E-02	6.98E-02	6.95E-02
$2s^2 2p^2 \ ^3P$	$2s^2 2p^3 3d \ ^3P$	1	3	1.03E-01	1.01E-01	9.05E-02	8.83E-02
$2s^2 2p^2 \ ^3P$	$2s^2 2p^3 3d \ ^3P$	3	3	3.19E-02	3.12E-02	2.82E-02	2.76E-02
$2s^2 2p^2 \ ^3P$	$2s^2 2p^3 3d \ ^1P$	1	3	2.77E-04	2.72E-04	4.00E-04	4.00E-04
$2s^2 2p^2 \ ^3P$	$2s^2 2p^3 3d \ ^1P$	3	3	3.60E-06	3.47E-06	.....	.....
$2s^2 2p^2 \ ^1S$	$2s 2p^3 \ ^3D$	1	3	2.75E-07	3.25E-07	.....	.....
$2s^2 2p^2 \ ^1S$	$2s 2p^3 \ ^3P$	1	3	1.07E-06	2.47E-06	.....	.....
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3s \ ^3P$	1	3	3.45E-04	2.76E-04	5.00E-04	3.00E-04
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3s \ ^1P$	1	3	3.08E-02	2.42E-02	9.30E-03	5.70E-03
$2s^2 2p^2 \ ^1S$	$2s 2p^3 \ ^3S$	1	3	2.51E-06	2.75E-06	.....	.....
$2s^2 2p^2 \ ^1S$	$2s 2p^3 \ ^1P$	1	3	2.49E-01	2.92E-01	3.13E-01	3.51E-01
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3d \ ^3D$	1	3	2.04E-04	2.03E-04	2.00E-04	2.00E-04
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3d \ ^3P$	1	3	2.11E-04	2.10E-04	3.00E-04	3.00E-04
$2s^2 2p^2 \ ^1S$	$2s^2 2p 3d \ ^1P$	1	3	5.60E-01	5.58E-01	4.27E-01	4.13E-01
$2s^2 2p 3p \ ^3P$	$2s^2 2p 3d \ ^3P$	1	3	1.16E-01	1.09E-01	1.21E-01	1.15E-01
$2s^2 2p 3p \ ^3P$	$2s^2 2p 3d \ ^1P$	1	3	1.54E-04	1.58E-04	3.00E-04	3.00E-04
$2s^2 2p 3p \ ^3P$	$2s^2 2p 3d \ ^3D$	1	3	4.92E-01	4.67E-01	5.07E-01	4.97E-01



$2s^2 2p^3 p$	$^1S$	$2s^2 2p^3 d$	$^3D$	1	3	9.84E-05	7.61E-05	2.00E-04	2.00E-04
$2s^2 2p^3 p$	$^1S$	$2s^2 2p^3 d$	$^3P$	1	3	1.53E-04	1.26E-04	4.00E-04	4.00E-04
$2s^2 2p^3 p$	$^1S$	$2s^2 2p^3 d$	$^1P$	1	3	6.50E-01	5.72E-01	7.09E-01	6.96E-01
$2s 2p^3$	$^3D$	$2s^2 2p^3 p$	$^3D$	3	3	1.58E-03	1.56E-03	1.40E-03	1.30E-03
$2s 2p^3$	$^3P$	$2s^2 2p^3 p$	$^3D$	3	3	6.38E-03	5.57E-03	2.80E-03	2.50E-03
$2s 2p^3$	$^3P$	$2s^2 2p^3 p$	$^3D$	1	3	2.60E-02	2.26E-02	1.93E-02	1.72E-02

<sup>a</sup> K. L. Bell, A. Hibbert and R. P. Stafford, *Physica Scripta*, (1995).

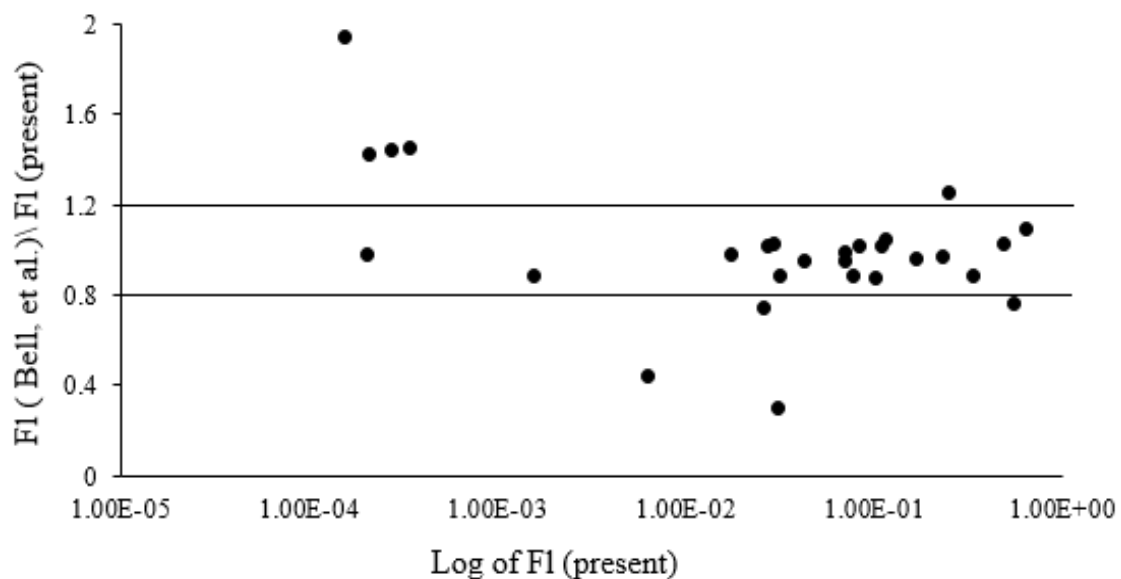


FIG 6. The ratio between length of Bell *et al* and the present oscillator strengths as a function of length form of the present oscillator strengths for some dipole E1 transitions has been shown. The solid lines represent a 20% deviation.

## CHAPTER IV

### CONCLUSION

A number of optical, ultraviolet, and extreme ultraviolet features of N II have been observed in the emission spectra of Titan's upper atmosphere, Saturn's inner magnetosphere, Sun, and other astrophysical objects. Also, several N II lines are important to our understanding of elemental abundance, density, and temperature of astrophysical plasmas. We have presented our calculation of excitation energy states, lifetimes, oscillator strengths, and transition probabilities. The calculations have been obtained using the HF and MCHF methods. Also, all calculations were based on the semi-relativistic approach based on the Breit-Pauli Hamiltonian. We have compared our calculation with the data from NIST, Bell *et al*, Tayal, and Froese Fischer. The data from Froese Fischer follow the same MCHF semi-relativistic approach for N II that we used in our calculations. The general agreement between our results and other computational results is satisfactory, except for some transitions where there are significant discrepancies. The discrepancies are especially larger for some weak semi-forbidden transitions with small oscillator strengths due to cancellation effects.

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